Bayesian inference for heteroscedastic Rician time series with applications to fMRI

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August 28, 2014

Overview

- Functional Magnetic Resonance Imaging (fMRI)
- The BOLD signal measures brain activity
- Rician likelihood function for the magnitude of the BOLD signals
- Bayesian inference using MCMC
- Activation patterns for our proposed Rician model and a corresponding Gaussian model
- Conclusions

Functional Magnetic Resonance Imaging (fMRI)

- fMRI is a non-invasive neuroimaging technique that measures brain activity. An active brain area consumes more oxygenated blood to support neural activity.
- The blood oxygen level-dependent (BOLD) signal measures changes in blood flows and is an indirect measure of neural activity.
- The brain is divided into a set of “cubes”. Our data from a certain multi-sensoring task paradigm contains “cubes” with dimensions 1.6 mm$\times$ 1.6 mm$\times$ 1.8 mm.
- The BOLD signal was measured every third second (TR = 3 s.), giving a time series of 111 BOLD signals in each out of 573,440 voxels.

Distribution of the magnitude of the BOLD signal

- The BOLD signal at time t is complex-valued:

$$\tilde{y} = a_t + b_t i,$$

$a_t \sim N(\mu_t \cos\theta_t, \sigma_t^2)$ indep. of $b_t \sim N(\mu_t \sin\theta_t, \sigma_t^2)$.

- The magnitude of $\tilde{y}$ is $y_t = |\tilde{y}| = \sqrt{a_t^2 + b_t^2}$, which follows a Rician distribution (Gudbjartsson and Patz, 1995)

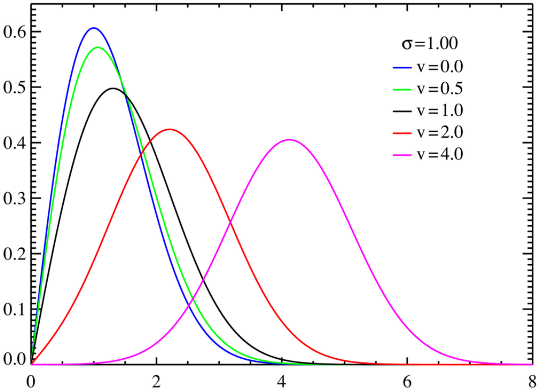
$$y_t | \mu_t, \sigma_t^2 \sim Rice(\mu_t, \sigma_t^2)$$

with density function

$$p\left(y_t | \mu_t, \sigma_t^2\right) = \frac{y_t}{\sigma_t^2} \exp\left(-\frac{\left(y_t^2 + \mu_t^2\right)}{2\sigma_t^2}\right) I_0\left(\frac{y_t \mu_t}{\sigma_t^2}\right)$$

where $I_0(\cdot)$ is the modified bessel function of the first kind with order zero.

Rice distribution



Modeling the BOLD signal: Rician versus Gaussian

- The common practice in fMRI studies is to assume that the magnitude of the BOLD signal follows a linear model with Gaussian noise.
- The widespread use of the Gaussian distribution is due to simpler statistical properties and that the Rician distribution is well approximated by a Gaussian distribution when the Signal-to-Noise Ratio (SNR) is high.
- We use the following definition of SNR: $SNR = \frac{\bar{\mu}_t}{\bar{\sigma}_t}$. For $SNR > 3$ the Gaussian distribution approximates the Rician distribution well.
- The push toward higher spatial resolutions (smaller voxels) in fMRI studies (Feinberg and Yacoub, 2012) may reduce the SNRs (Macovski, 1996) to a level where inaccurate Gaussian approximations lead to severely distorted activation maps.

The Rician likelihood function

- We propose a Rician model that allows for both temporal dependence of the $(y_t)_{t=1}^{T}$ sequence and heteroscedasticity in the noise:

$$y_t | y_t^{(k)}, \mu_t, \sigma_t^2 \sim Rice(\mu_t, \sigma_t^2)$$
$$\mu_t = \beta_0 + x_t' \beta$$
$$\ln \sigma_t^2 = \alpha_0 + z_t' \alpha$$

  where $y_t^{(k)} = (y_{t-1}, \ldots, y_{t-k})$ is the time lags of y_t up to k time periods back, and both x_t and z_t may include elements of $y_t^{(k)}$ in addition to time-varying drift components and the predicted BOLD signal components for functional activity.

- The predicted BOLD signal at time t for functional activity, $B(t)$, is the convolution between the hemodynamic response function, $h(\cdot)$, and the neural activation, $N(\cdot)$, in a time-invariant linear system: $B(t) = \int N(\tau) h(t-\tau)\, d\tau$.

- The Rician likelihood function: $L_R = \prod_{t=k+1}^{T} p\left(y_t | y_t^{(k)}, \mu_t, \sigma_t^2\right)$

Bayesian inference using MCMC

- The Rician distribution does not belong to the exponential family, which makes traditional likelihood-based inference problematic.
- We utilize a general Bayesian approach in Villani et al. (2009) and develop a highly efficient MCMC scheme to explore the joint posterior distribution of all model parameters, including the activation.
- We allow for Bayesian variable selection so that the relevant regressors and lags are chosen automatically in the MCMC sampling.
- Our MCMC proposal distribution is tailored directly to the observed likelihood for the Rician model, which leads to fast mixing and high acceptance probabilities in the MCMC.
- The high numerical efficiency is absolutely crucial for fast processing of the large number of voxels in a typical fMRI study.
- The prior distributions on the model parameters are well designed unit information priors based on the Fisher information matrix obtained by simulation (Villani et al. 2012).

Comparing activation patterns

- We compare the proposed Rician model to a corresponding Gaussian model on simulated data with Rician noise at different SNR levels.
- The baseline data generating processes mimics the results from the multi-sensoring task fMRI experiment (1.6 mm$\times$ 1.6 mm$\times$ 1.8 mm), but with the noise variance manipulated to experimentally control the SNR levels in the simulated datasets.
- Both of the compared models are analyzed by the same Bayesian methods with the same unit prior information on the model parameters.

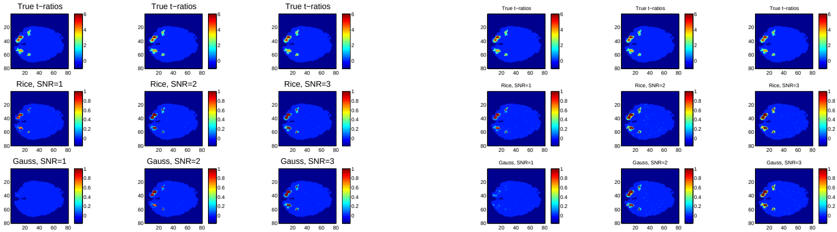


Figure 1. Comparison of activation inferences in the Rician and Gaussian models on simulated data. Top row: True activations in the Rician data generating model. Middle (Rice) and bottom (Gauss) row: Percentage of simulated datasets where the posterior probability of activation is larger than 99 % in the left part of the graphs and larger than 95 % in the right part of the graphs.

Conclusions

- We propose a new heteroscedastic Rician time series model, and an efficient MCMC algorithm for Bayesian inference with automatic variable selection.
- We use simulated fMRI data to show that the Rician model correctly locates the activated brain regions with a very high accuracy.
- The Gaussian model can completely miss out on whole activated regions when the SNR is small.

References

- Feinberg, D. et al. (2012), 'The rapid development of high speed, resolution and precision in fMRI', NeuroImage vol. 62, 720-725.
- Gudbjartsson, H. et al. (1995), 'The Rician distribution of noisy MRI data', Magnetic Resonance in Medicine, vol 34, 910-914.
- Macovski, A. (1996), 'Noise in MRI', Magnetic Resonance in Medicine, vol 36, 494-497.
- Villani, M. et al. (2009), 'Regression density estimation using smooth adaptive Gaussian mixtures', Journal of Econometrics, vol. 153, 155-173.
- Villani, M. et al. (2012), 'Generalized smooth finite mixtures', Journal of Econometrics, vol. 171, 121-133.
- Zhu, H. et al. (2009) 'Regression models for identifying noise sources in magnetic resonance images', Journal of the American Statistical Association, vol. 104, 623-637.